Dynamics of Spin Relaxation near the Edge of Two-Dimensional Electron Gas

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We report calculations of spin relaxation dynamics of two-dimensional electron gas with spin-orbit interaction at the edge region. It is found that the relaxation of spin polarization near the edge is more slow than relaxation in the bulk. That results finally in the spin accumulation at the edge. Time dependence of spin polarization density is calculated analytically and numerically. The mechanism of slower spin relaxation near the edge is related to electrons reflections from the boundary and the lack of the translation symmetry. These reflections partially compensate electron spin precession generated by spin-orbit interaction, consequently making the spin polarization near the edge long living. This effect is accompanied by spin polarization oscillations and spin polarization transfer from the perpendicular to in-plane component.

The prospects for creating a semiconductor-based spintronic device [1, 2, 3, 4, 5, 6] have generated an emphasis on the studies of properties of electron spin polarization in semiconductor nanostructures [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]. Great interest has been expressed in dynamics of electron spin polarization [7, 8, 9, 10, 11, 12]. For instance, spin relaxation dynamics has been studied in two-dimensional electron gas (2DEG) [7, 8], two-dimensional channels [9, 10], open Sinai billiards (2DEG with a lattice of antidots) [11], and ballistic quantum dots [12]. It was found that the sample geometry [11] as well as specific initial conditions [7, 8] could have a significant effect on electron spin relaxation.

D'yakonov-Perel' (DP) spin relaxation mechanism [18] is the leading spin relaxation mechanism in many important experimental situations. In the framework of DP theory, initially homogeneous electron spin polarization exponentially relaxes to zero (or to some finite equilibrium value) with time. However, DP theory was formulated for the bulk of a sample. Considering electron spin relaxation near the edge of 2DEG, one would expect the same relaxation scenario. This expectation, however, is not correct. We demonstrate in this Letter that the spin relaxation dynamics near the edge is rather unusual and can not be described by a simple exponential law, as follows from the DP theory. We observe a longer spin relaxation time near the edge, spin polarization oscillations and spin polarization transfer from the perpendicular (to 2DEG) to in-plane component.

Let us consider a two-dimensional electron gas with the Rashba spin-orbit interaction [19], which couples electron space and spin degrees of freedom:

$$H_R = \alpha \hbar^{-1} \left(\sigma_x p_y - \sigma_y p_x \right), \qquad (1)$$

where α is a constant, σ is the Pauli matrix vector corresponding to the electron spin, and $\mathbf{p}$ is the momentum of the electron confined in two-dimensional geometry. From the point of view of electron spin, the effect of spin-orbit interaction can be regarded as an effective magnetic field

acting on electron spin. Momentum scatterings reorient the direction of this field, thus leading to average spin relaxation (DP relaxation). Intuitively, electron reflections from the 2DEG edges should increase spin relaxation time, since electron spin rotations will be partially compensated. In order to study this phenomena, we use both analytical and numerical approaches.

Let us consider a two-dimensional electron gas occupying half-space $x > 0$ (Fig. 1) with electron spins homogeneously polarized in z direction, perpendicular to 2DEG, in the initial moment of time $t = 0$. From the experimental point of view, the initial electron spin polarization as well as detection of spin polarization can be realized using the optical technique [20] or any other appropriate method. It is convenient to describe the spin state of i-th electron via the spin polarization vector $\mathbf{P}_i = Tr(\rho_i \sigma)$, where ρ_i is the single-particle density matrix [21]. In the framework of drift-diffusion approximation, the ini-
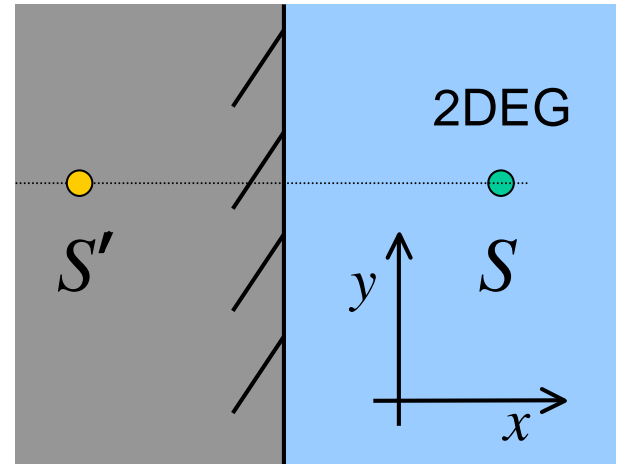

FIG. 1: Schematic of the system: 2DEG occupies half-plane $x > 0$. The influence of the boundary located at $x = 0$ on the electron space motion is taken into account by introducing the mirror source S'.

tial dynamics of the spin polarization density $\mathbf{P}$, which is defined via $\mathbf{P}d^2r = \sum_{i \in d^2r} \mathbf{P}_i$ (here $d^2r = dxdy$ is a space volume), can be calculated using the following expression

$$\mathbf{P}(x,y,t) = \int_0^\infty \int_{-\infty}^\infty G(x-x', y-y', t)\mathbf{P}'_{(x,y),(x',y')} dy' dx',$$

(2)

where $G(x-x', y-y', t)$ is the diffusion Green function (solution of a diffusion equation with a point source), and $\mathbf{P}'_{(x,y),(x',y')}$ represents a contribution of the initial spin polarization density at point (x', y') to $\mathbf{P}(x, y, t)$. The structure of Eq. (2) can be easily understood. Electron spin polarization density in a space volume with coordinates (x, y) at a selected moment of time t is given by a sum of spin polarization vectors of all electrons located in this volume. The diffusion Green function $G(x-x', y-y', t)$ gives probability for electrons to diffuse from the point (x', y') to (x, y), while $\mathbf{P}'_{(x,y),(x',y')}$ describes the spin polarization of these electrons.

We note that Eq. (2) governs only the initial spin relaxation dynamics. The main approximation behind Eq. (2) is the assumption that different spin rotations commute with each other, and spin precession angle φ is proportional to the distance between (x', y') and (x, y). This assumption is justified when the spin precession angle per mean free path is small and the time is short. Moreover, it is assumed that evolution of electron spin degree of freedom is superimposed on the space motion of the charge carriers. In other word, the influence of the spin-orbit interaction on electron space motion is neglected. If $\mathbf{a}$ is the unit vector along the precession axis, then [13]

$$\mathbf{P}'_{(x,y),(x',y')} = \mathbf{P} + \mathbf{P}_\perp (\cos\varphi - 1) + \mathbf{a} \times \mathbf{P}\sin\varphi, \quad (3)$$

where $\mathbf{P}_\perp = \mathbf{P} - \mathbf{a}(\mathbf{aP})$ is the component of the spin polarization perpendicular to the precession axis, $\varphi = \eta r$, η is the spin precession angle per unit length, $\mathbf{r} = (x-x', y-y')$, $r = |\mathbf{r}|$, $\mathbf{a} = \hat{z} \times \mathbf{r}/r$, and $\hat{z}$ is the unit vector in z direction. We emphasize that the Rashba spin-orbit interaction is the origin of spin polarization rotations described by Eq. (3). The coupling constant α of the Rashba interaction enters into our model through the parameter η (for details see Ref. [10]).

When 2DEG occupies only a half-space, the Green function $G(x-x', y-y', t)$ that appears in Eq. (2) can be obtained using mirror-image approach. Within this approach, the Green function $G(x-x', y-y', t)$ is written as a superposition of two point-source solutions

$$G = G_0(x-x', y-y', t) + G_0(x-x', y+y', t), \quad (4)$$

where $G_0(\mathbf{r}, t) = 1/(4\pi Dt)e^{-\mathbf{r}^2/(4Dt)}$ is the full space Green function, $D = L_p^2/(2\tau_p)$ is the diffusion coefficient,
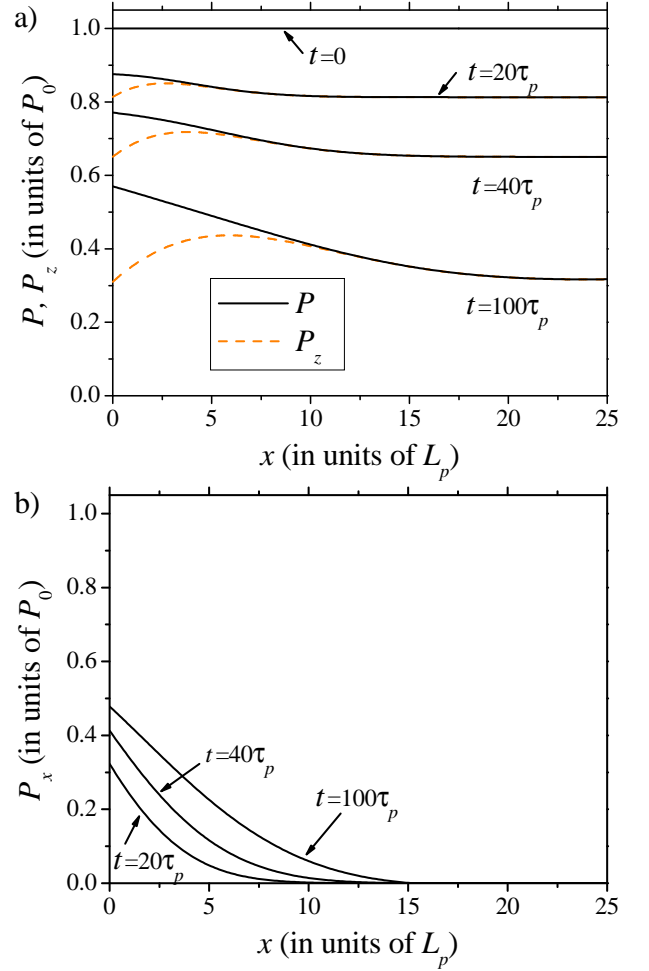

FIG. 2: (Color online) Initial evolution of spin polarization near the edge calculated within the drift-diffusion approximation. Relaxation of electron spins occurs slowly near the edge. All graphs presented in this Letter were obtained using the same parameter $\eta L_p = 0.1$.

L_p is the mean free path, and τ_p is the momentum relaxation time. While the first term in the right hand side in Eq. (4) describes the real source, the second term is a so-called mirror source (Fig. 1), that is introduced to describe the reflections of electrons from the boundary.

Substituting Eq. (3) into Eq. (2) and taking the initial spin polarization in the form $\mathbf{P}(t = 0) = P_0\hat{z}$, we can obtain the following expressions for the components of the spin polarization density:

$$P_x = P_0 \int_0^\infty \int_{-\infty}^\infty G(x-x', y-y', t)\frac{x'-x}{r}\sin(\eta r)dy'dx'$$

(5)

$$P_y = 0$$

(6)

$$P_z = P_0 \int\limits_0^\infty \int\limits_{-\infty}^\infty G(x - x', y - y', t) \cos(\eta r) dy' dx'. \quad (7)$$

It is obvious that $P_y = 0$ from symmetry considerations. We can not perform the integration in Eqs. (5) and (7) in the general form. However, these integrals can be evaluated explicitly for short times. In this case, since the exponents of the Green function effectively cut off contributions of the terms with large r, the geometric functions in Eqs. (5), (7) can be approximated as $\sin(\eta r) \approx \eta r$ and $\cos(\eta r) \approx 1 - (\eta r)^2/2$. Then, after the integration, we get

$$\frac{P_x}{P_0} = \eta x \left(1 - Erf\left[\frac{x}{\sqrt{4Dt}} \right] \right) - \eta \sqrt{\frac{4Dt}{\pi}} e^{-\frac{x^2}{4Dt}} \quad (8)$$

and

$$\frac{P_z}{P_0} = 1 - \eta^2 x^2 \left(1 - Erf\left[\frac{x}{\sqrt{4Dt}} \right] \right) +$$
$$2Dt\eta^2 \left(-1 + \frac{xe^{-\frac{x^2}{4Dt}}}{\sqrt{\pi Dt}} \right), \quad (9)$$

where, $Erf[z]$ is the error function, given by $Erf[z] = (2/\sqrt{\pi}) \int_0^z e^{-t^2} dt$. It follows from Eqs. (8) and (9) that the area of nonhomogeneous spin polarization density near the edge increases in time and is characterized by the length $l \sim \sqrt{4Dt}$. The behavior of spin relaxation in the bulk region is governed by the standard DP theory: $P_x = 0$ and the relaxation of P_z is described by the DP relaxation time $\tau_r = \tau_p/(L_p\eta)^2$. Let us estimate the limits of applicability of Eqs. (8), (9). The characteristic length scale of the Green functions is $\sqrt{4Dt}$, while the characteristic length of the geometric functions is η^{-1}. Setting these two lengths equal we find that Eqs. (8), (9) are valid while $t \lesssim 1/(4D\eta^2)$.

Figure 2 shows the spin polarization density $P = \sqrt{P_x^2 + P_z^2}$ and its components P_x and P_z ($P_y = 0$ accordingly to Eq. (6)) as a function of x at several moments of time. Spin polarization density represented in Fig. 2 was obtained using a numerical integration in Eqs. (5) and (7). It is clearly seen the absolute value of the spin polarization density P is higher in the edge region. It is interesting to note the the higher spin polarization density near the edge is largely due to the x component of spin polarization density (Fig. 2b). From Eq. (5) it follows that initial growth of P_x at $x = 0$ is proportional to $\sqrt{t}$. We also observe formation of a local maximum of P_z in the vicinity of the edge. The peak of P_z drifts from $x = 0$ with time.

In order to check the results of our drift-diffusion approximation as well as to find long time behavior of spin polarization density, we study the dynamics of spin relaxation near the edge using a Monte Carlo simulation
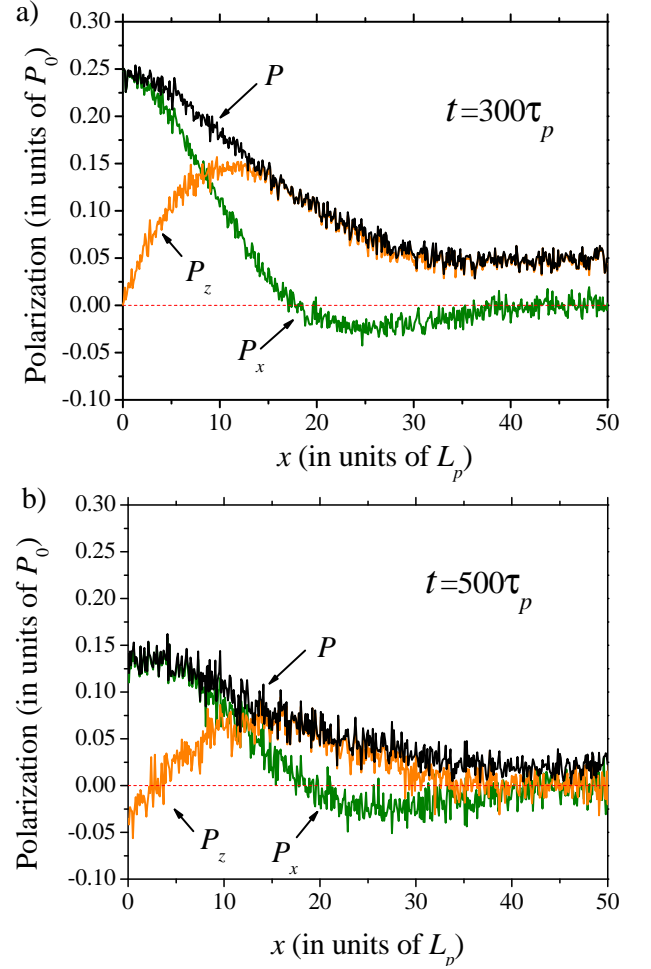

FIG. 3: (Color online) Spin polarization near the edge at longer times. Suppression of spin relaxation near the edge, as well as spin oscillations in time and space are observed. The dotted red line is for the eye.

scheme that was proposed in Ref. [10] and subsequently used in Refs. [7, 11]. Within the Monte Carlo simulation algorithm, the space motion of 2DEG electrons is considered to be along classical (linear) trajectories interrupted by the bulk scattering events. Our modeling involves spin-independent bulk scattering processes, which could be caused, e.g., by impurities. For the sake of simplicity, the scattering due to such events is assumed to be elastic and isotropic, i.e., the magnitude of the electron velocity is conserved in the scattering, while the final direction of the velocity vector is randomly selected. The time scale of the bulk scattering events can then be fully characterized by a single rate parameter [10], the momentum relaxation time, τ_p. It is connected to the mean free path by $L_p = v\tau_p$. Here v is the mean electron velocity. We use reflecting boundary conditions from the edge: the longitudinal component of the electron velocity is preserved and the sign of the normal component is changed in collisions. The typical number of electrons used in our simulations is $2 \cdot 10^7$.

We have found that the results of Monte Carlo simulations at short times are in perfect agreement with predictions of drift-diffusion model. Fig. 3 shows spin polarization density obtained via Monte Carlo simulations at longer times. At these times the space distribution of spin polarization is nontrivial and the slowing down of the spin relaxation near the edge becomes more pronounced. Fig. 3 indicates the change of the sign of P_z at $x = 0$. The change of the sign of P_x occurs some distance away from the edge. Relaxation of spin polarization density in the bulk is in agreement with DP theory.

In conclusion, we have studied the spin relaxation dynamics in a 2DEG near the edge. The obtained result indicates that the spin relaxation in the edge region deviates from the D'yakonov-Perel' theory. Specifically, spin relaxation occurs slowly in the edge region and is accompanied by spin oscillations in time and space and spin polarization transfer from perpendicular to in-plane component. We have derived an analytical formula describing initial spin polarization dynamics. Long time behaviour of spin polarization density was studied numerically. Qualitatively, the observed effects are related to the lack of translation symmetry in the system. Reflections of electron from the boundary partially compensate spin precessions due to spin-orbit interaction. Moreover, the lack of the actual electron flow from the half-plane $x < 0$ gives rise to non-zero in-plane component of spin polarization density near the edge, since this component of spin polarization for electrons coming from $x > 0$ into the edge region is uncompensated. In some sense the structure that forms near the edge is similar to long-living spin coherence states previously discussed in Ref. [7]. We note that our results are not directly related to the spin Hall effect [22], in which the electron spins become polarized in an applied electric field. Nevertheless, our results could be important in interpretation of spin Hall experiments.

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